

Funding Public Goods Through Dedicated Taxes on Private Goods

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Abstract

This paper examines positive and normative consequences of dedicated taxes, which entail taxing a private good in order to finance the provision of a public good. Our approach differs from the classic public finance literature because of its setup in a game-theoretic model of private provision of an impure public good. We begin by showing that imposition of a dedicated tax can either increase or decrease demand for the taxed good. We then derive conditions showing why the optimal dedicated tax cannot, in general, achieve the Pareto optimal allocation. It can, however, generate a conditionally efficient equilibrium with comparatively more or less of the public good, depending, in part, on whether the public good and the taxed private good are Hicksian complements or substitutes. We also show a neutrality result: when individuals have the opportunity to make direct donations, dedicated taxes that are sufficiently low will leave the equilibrium allocation unchanged.

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1 Introduction

This paper examines potential advantages and disadvantages of financing the provision of a public good by taxing a related private good. Within the theoretical literature on financing public goods, common mechanisms that rely on centralized coordination are income or wealth taxes (often lump sum) or subsidies on private provision. Other approaches rely on the benefits principle, which suggests that individuals who benefit more from the public good should pay more for its provision. Toll roads provide a common example. Another example is that visitors to National Parks pay more through admission fees.¹ There are, however, reasons why the direct benefits principle might not be desirable in many contexts, including distributional equity and administrative feasibility.

These concerns often lead to ideas about taxing related goods, a notion we refer to as a dedicated tax throughout the present paper. For example, in lieu of monitoring the distance that drivers travel on public roads for purposes of taxation, gasoline taxes are often used to finance transportation infrastructure. Dedicated taxes are also considered in the context of parks and public lands. Rather than charging high admission fees, public lands and parks can be financed to some degree through taxes on related goods, such as “gear taxes” on outdoor equipment and hunting licenses.² The intuitive appeal underlying such policies and proposals is that taxing seemingly related goods or services has advantages for financing public goods. In what follows, we provide a theoretical analysis to evaluate this intuition. In doing so, we develop an approach for examining the positive and normative consequences of using dedicated taxes to finance public goods.

Our analysis is related to the seminal literature in public finance on the optimal supply of public goods when financed through distortionary taxes (e.g., Diamond and Mirrlees, 1971; Stiglitz and Dasgupta, 1971; Atkinson and Stern, 1974). Part of our

¹Included in this volume are papers by Ji et al. (2020) and Lupi et al. (2020) that provide empirical analyses of user fees for public lakes and beaches, respectively.

²Papers by Banzhaf and Smith (2020) and Walls and Ashenfarb (2020), which are also in this volume, provide background on the history of funding for public lands in the United States, including a discussion of excises on hunting and fishing gear and proposals for broader based gear taxes.

contribution is to show results in the context of an impure public good model. By explicitly linking the consumption of a private good with provision of a public good, dedicated taxes create an impure public good similar to that analyzed by Cornes and Sandler (1984; 1994; 1996).³ Using this framework, we are able to show in a direct and transparent way the incentives that dedicated taxes create, their efficiency consequences, and their potential scope for financing the provision of public goods.

We establish four main results. First, we show the conditions under which imposition of a dedicated tax can either increase or decrease demand for the taxed good. Second, we derive intuitive conditions showing how the Nash equilibrium under an optimal dedicated tax cannot achieve the Pareto optimal allocation, except in the limiting case of a single agent, which is equivalent to assuming there is no public good. Part of the reason is that the dedicated tax does not eliminate the free-riding incentive, which affects consumption of the both the taxed private good and the public good. Third, we show that the equilibrium level of the public good can exceed or fall short of the Pareto optimal level, depending, in part, on whether the public good and taxed private good are Hicksian complements or substitutes, respectively. Finally, we show a neutrality result whereby dedicated taxes that are sufficiently low will have no effect on the equilibrium allocation if individuals have the opportunity to simultaneously make voluntary contributions.

The remainder of the paper is organized as follows. The next section introduces our analytical approach of using virtual prices and income to establish comparative static results. We show how changes in the exogenously given level of a public good affects demand for private goods. Section 3 works through the Pareto efficient benchmark and its implementability with lump-sum taxes. Section 4 considers imposition of a dedicated tax and its consequences for individual behavior and the existence and uniqueness of a Nash equilibrium. Section 5 analyzes properties of the optimal dedicated tax and compares the associated equilibrium to the Pareto efficient allocation. Section 6 generalizes the setup to allow for the possibility of direct donations and

³Earlier papers on related topics include Brownlee's (1961) discussion of funding public goods and services at least partially through sales receipts and subsequent formalizations of the idea by Cicchetti and Smith (1970) and Holtermann (1972).

establishes the neutrality result. Section 7 provides a concluding discussion.

2 Private and Public Goods

To establish some preliminary intuition, we begin with a basic utility maximization problem where a representative individual chooses consumption of private goods while taking the level of a public good as exogenously given. In particular, individual i solves

$$\max_{x_1^i, x_2^i} U^i(x_1^i, x_2^i, X_3) \text{ s.t. } p_1 x_1^i + p_2 x_2^i = w^i \text{ and } X_3 = \tilde{X}_3, \quad (1)$$

where x_1^i and x_2^i are private goods with the respective prices, w^i is the individual's wealth, and X_3 is a public good that is exogenously provided at level \tilde{X}_3 .⁴ We assume for the time being there is no opportunity for individuals to privately provide the public good. This assumption means that the constraint $X_3 = \tilde{X}_3$ is redundant, but we nevertheless include it in the statement of the problem with an eye towards the generalizations we consider later in the paper. In all cases, we will use \tilde{X}_3 to denote public good provision that is taken as exogenous by the agent in question. Because we are focusing initially on a single individual, we drop superscripts for now. The solution to (1) can be written as demand functions $\hat{x}_j(p_1, p_2, \tilde{X}_3, w)$ for good $j = 1, 2$.

Although the basic setup does not allow individuals to privately provide the public good, we can derive the individual's marginal willingness to pay (WTP) for X_3 . Solving the dual of (1) yields an expenditure function $e(p_1, p_2, \tilde{X}_3, U^0) = w$. It follows that the individual's marginal WTP for the public good, denoted π_3 , will itself be a function of the exogenous parameters and satisfy

$$\pi_3(p_1, p_2, \tilde{X}_3, w) = \frac{\partial e(p_1, p_2, \tilde{X}_3, U^0)}{\partial \tilde{X}_3}. \quad (2)$$

This expression indicates how marginal WTP is equal to the compensating change in income for a marginal change in the quantity of the public good.

⁴The setup builds on the modeling tradition in the context of privately provided pure and impure public goods, which assumes a fixed endowment of wealth and thereby abstracts from the labor/leisure choice that is central to much of the literature on optimal taxation.

We now consider how changes in the exogenously provided level of the public good affects demand for the private goods. That is, we are interested in what determines the sign of $\partial \hat{x}_j / \partial \tilde{X}_3$. While these results are interesting in their own right, the approach that we employ for showing them helps provide the basis for the methods we employ in subsequent sections.

We derive the comparative static results in terms of familiar price and income effects using notations of virtual prices and income.⁵ The first step is to consider an alternative utility maximization problem where the individual can choose the aggregate level of the public good X_3 at a price equal to π_3 , in addition to the private goods. For the moment, π_3 need not equal that defined in equation (2), but the connection will soon become clear. In this case, the individual's budget constraint can be rewritten as $p_1 x_1 + p_2 x_2 + \pi_3 X_3 = w + \pi_3 \tilde{X}_3$, where the right-hand side is the individual's virtual full income and represents her endowment plus the value of public good spill-ins. Let $\mu = w + \pi_3 \tilde{X}_3$ denote full income. By implicitly choosing π_3 , we can satisfy the following condition for $j = 1, 2$:

$$\hat{x}_j(p_1, p_2, w, \tilde{X}_3) = x_j(p_1, p_2, \pi_3(\bullet), \mu(\bullet)). \quad (3)$$

This means that the solution to the “unrestricted” utility maximization problem, written in terms of demand for the private goods, will be identical to that for (1). For purposes of clarity, recall that demand functions with a circumflex (or hat) denote solutions to the “restricted” utility maximization problem (1), whereas those without the additional notation on the right-hand side of (3) are the unrestricted solutions.

Satisfying (3) for both private goods also means that demand for the public good will be a knife-edged solution right at the corner such that

$$\tilde{X}_3 = X_3(p_1, p_2, \pi_3(\bullet), \mu(\bullet)), \quad (4)$$

where the upper-case X_3 denotes demand for the aggregate level of the public good,

⁵This is the approach used by Cornes and Sandler (1994; 1996) in their study of the impure public good model, but an earlier analysis that employs the same general approach for comparative static analysis of private and rationed goods is found in Madden (1991).

which is equal to the exogenously provided level given in (1). Now, it is worth clearly stating that the value of π_3 that satisfies (3) is equal to the marginal WTP defined in (2). Nevertheless, it is also worth pointing out the former is a Marshallian measure and the latter is a Hicksian measure, so the two will diverge with non-marginal changes from the initial allocation at which they are defined.⁶

Differentiating (3) with respect to \tilde{X}_3 produces the comparative statics of interest, where marginal changes in the restricted demand functions can be written in terms of changes in the unrestricted demand functions:

$$\begin{aligned} \frac{\partial \hat{x}_j(p_1, p_2, w, \tilde{X}_3)}{\partial \tilde{X}_3} &= \frac{\partial x_j(p_1, p_2, \pi_3(\cdot), \mu(\cdot))}{\partial \tilde{X}_3} \\ &= x_{j3} \frac{\partial \pi_3}{\partial \tilde{X}_3} + x_{j\mu} \frac{\partial \mu}{\partial \tilde{X}_3} \\ &= [C_{j3} - X_3 x_{j\mu}] \frac{\partial \pi_3}{\partial \tilde{X}_3} + x_{j\mu} \frac{\partial \mu}{\partial \tilde{X}_3}. \end{aligned} \quad (5)$$

As before, j indexes the good in question. Additional subscripts $k = 1, 2, 3$ represent partial derivatives with respect to the (virtual) prices of goods x_1 , x_2 , and X_3 respectively, while the subscript μ denotes the partial derivative with respect to virtual income. The last equality comes from substituting in the Slutsky equation, and C_{jk} denotes the compensated (Hicksian) demand response for good $j = 1, 2, 3$ with respect to a change in price $k = 1, 2, 3$. Notice that this equation expresses the results in standard price and income responses for a familiar (unrestricted) problem.

The only things that remain to be solved for are changes in the virtual magnitudes π_3 and μ with respect to a change in \tilde{X}_3 . Following the procedure described by Cornes and Sandler (1996), this is possible using Cramer's rule and the identifying equations for (i) the budget constraint and (ii) the level of the public good. We provide the details and solutions in Appendix A.1, which can be substituted into (5) to yield

$$\frac{\partial \hat{x}_j}{\partial \tilde{X}_3} = \frac{C_{j3}(1 - \pi_3 X_{3\mu})}{C_{33}} + \pi_3 x_{j\mu}. \quad (6)$$

In this equation, the term $X_{3\mu}$ captures how the individual's unrestricted demand for

⁶See Neary and Roberts (1980) for a related discussion.

aggregate X_3 responds to a change in virtual income, while $x_{j\mu}$ denotes the income effect on demand for private goods $j = 1, 2$.

Several results follow. If all goods are normal in the usual unrestricted sense (i.e., $X_{3\mu} > 0$ and $x_{j\mu} > 0$ for $j = 1, 2$), then the full income effects on demand are not only positive, but $1 - \pi_3 X_{3\mu} > 0$ because a unit increase in income must be spent on all goods. This means that (6) is positive if x_j and X_3 are Hicksian complements (i.e., $C_{j3} < 0$), whereas the sign is indeterminate if the two goods are Hicksian substitutes (i.e., $C_{j3} > 0$). With the special case of quasilinear preferences of the form $x_k + F(x_j, X_3)$, equation (6) is positive or negative if x_j and the public good are Hicksian complements or substitutes, respectively.⁷ Together, these results show how demand for a private good changes with a change in the level of an exogenously provided public good, and whether the goods are complements or substitutes plays a critical role.

3 Pareto Efficiency

We now consider an economy that consists of $n \geq 2$ individuals and solve for the efficient level of the public good. This provides an important point of comparison for our subsequent consideration of a dedicated tax mechanism. We assume the cost of providing the public good is unity, and without loss of generality, we normalize the level of the public good that does not come through private provision to zero. The aggregate level of the public good is thus defined as $X_3 = \sum_{i=1}^n x_3^i$.

Solving for the set of Pareto optimal allocations is a matter of standard practice in public economics. All of the efficient allocations will satisfy the following first-order conditions

$$\sum_{i=1}^n \frac{U_3^i}{U_j^i} = \frac{1}{p_j} \text{ for } j = 1, 2 \text{ and } \frac{U_1^i}{U_2^i} = \frac{p_1}{p_2} \text{ for } i = 1, \dots, n, \quad (7)$$

where for completeness we include the derivation in Appendix A.2. The first condi-

⁷These results provide a specific application of the findings in Madden (1991) about the symmetry of substitute and complement relationships between goods based on changes in prices or quantities of a rationed good, which in this case is the level of the public good.

tion is the well-known Samuelson condition, where the sum of the marginal rates of substitution between the public good and all private goods equals the corresponding price ratios. The second is the standard condition for private goods, where the marginal rates of substitution between goods equals the price ratio for all individuals.

We simplify things further by assuming identical preferences across all individuals and focusing on the symmetric allocation. The symmetry is helpful for our comparisons below and is also the allocation that a social planner would choose with equal welfare weights across individuals. With these assumptions, the unique solution will satisfy

$$\frac{nU_3}{U_j} = \frac{1}{p_j} \text{ for } j = 1, 2 \text{ and } \frac{U_1}{U_2} = \frac{p_1}{p_2}, \quad (8)$$

which is a special case of the conditions in (7). To be clear, this implies the same allocation of private goods for each individual, defined as (x_1^*, x_2^*) , where asterisks denote the solution for the social planner's problem. It also defines a unique level of the public good

$$X_3^* = \left(\sum_{i=1}^n w^i \right) - n(p_1 x_1^* + p_2 x_2^*). \quad (9)$$

We can verify the standard result that lump-sum taxes can be used to implement the Pareto optimal allocation. With individualized taxes τ_i , each individual's utility maximization problem is

$$\max_{x_1, x_2} U(x_1, x_2, X_3) \text{ s.t. } p_1 x_1 + p_2 x_2 = w_i - \tau_i \text{ and } X_3 = \sum_{i=1}^n \tau_i,$$

with the corresponding first-order condition

$$\frac{U_1(x_1, x_2, \sum_{i=1}^n \tau_i)}{U_2(x_1, x_2, \sum_{i=1}^n \tau_i)} = \frac{p_1}{p_2}. \quad (10)$$

Setting each individual's tax such that $\tau_i^* = w_i - p_1 x_1^* - p_2 x_2^*$ has two implications. The first is that $\sum_{i=1}^n \tau_i^* = X_3^*$ by (9), so the public good is fully funded. The second is that (x_1^*, x_2^*) satisfies each individual's budget constraint and (10), which is equivalent to the second condition in (8). This means that imposing τ_i^* for all i implements

the Pareto efficient and symmetric allocation as an equilibrium outcome. Note the possibility that τ_i^* can be a subsidy in some cases if an individual’s endowment is sufficiently low. To simplify things even further in what follows, we make the additional assumption of identical endowments w across individuals. In this case, the optimal lump-sum tax is a uniform “head” or “poll” tax that satisfies

$$\begin{aligned}\tau^* &= w - p_1 x_1^* + p_2 x_2^* \\ &= X_3^*/n.\end{aligned}$$

The Pareto optimal allocation, which we have verified can be implemented with lump-sum taxes, will provide a useful benchmark for our analysis that follows.

4 A Dedicated Tax

We now turn to a dedicated tax mechanism to finance the public good, which we assume applies to good x_2 without loss of generality. In particular, we model a tax rate of t_2 per unit x_2 , and all proceeds are used to finance the provision of X_3 .⁸ Examples of such dedicated taxes include the gear taxes to fund parks (mentioned previously) and real estate transfer taxes to fund the acquisition of open space lands. Other examples, which are not based on government provision, are a number of instances where private goods are bundled with contributions to public goods, such as the 1% For The Planet Program.⁹ In this section, we first characterize individual incentives before turning to the Nash equilibrium.

4.1 Individual Behavior

We begin with a representative individual’s utility maximization problem taking the exogenously given level of the public good \tilde{X}_3 , which now represents the provision of all others, as given. The individual’s problem can be written as

⁸We continue to assume that individuals are not able to make direct donations to provide the public good, though we relax this assumption in Section 6.

⁹See <https://www.onepercentfortheplanet.org/>.

$$\max_{x_1, x_2} U(x_1, x_2, t_2 x_2 + \tilde{X}_3) \text{ s.t. } w = p_1 x_1 + (p_2 + t_2) x_2. \quad (11)$$

Assuming an interior solution, the first-order conditions can be combined to

$$\frac{U_2 + t_2 U_3}{U_1} = \frac{p_2 + t_2}{p_1}. \quad (12)$$

This has an intuitive interpretation: the ratio of the marginal utilities (benefits) of the two goods equals the price ratio. The difference here from a more typical setup is that x_2 is linked to X_3 through t_2 , which defines the relative and constrained quantities and the tax-inclusive price. As shown in the numerator on the left-hand side, consumption of an additional unit of x_2 provides the marginal benefit of U_2 plus $t_2 U_3$. Suppressing notation for prices and w , we can fully characterize the solution to (11) as the function $\hat{x}_2(t_2, \tilde{X}_3)$. The choice of \hat{x}_1 is then defined through the budget constraint.

An interesting feature of the setup in (11) is that for any given level of t_2 , it is a special case of Cornes and Sandler's (1984; 1994; 1996) impure public good model. This follows because consumption of the taxed private good becomes associated with joint production of the public good. Nevertheless, the comparative static properties of the model will differ because here the price of the jointly produced good and the technology of joint production are not independent parameters, as they are both functions of t_2 .¹⁰

It is both interesting and useful to consider how demand for x_2 changes with imposition of the dedicated tax. Keep in mind this is not simply a price effect, as an increase in the tax simultaneously provides the public good. We again employ virtual prices and income to derive results in terms of familiar price and income effects. In

¹⁰In particular, using the notation in Cornes and Sandler (1994), the relationship between models follows by setting $\gamma = t_2$ and $p = p_2 + t_2$.

Appendix A.3, we derive the following general result:

$$\frac{\partial \hat{x}_2}{\partial t_2} = \frac{x_2(t_2 C_{22} - C_{23}) + (1 - \pi_3)\Psi + (1 - \pi_3)x_2 [(C_{33} - t_2 C_{32})x_{2\mu} + (t_2 C_{22} - C_{23})X_{3\mu}]}{\Omega} \quad (13)$$

where

$$\Psi = C_{23}C_{32} - C_{22}C_{33} < 0$$

and

$$\Omega = t_2(C_{23} + C_{32}) - t_2^2 C_{22} - C_{33} > 0,$$

and the signs of these latter two expressions follow by negative semi-definiteness of the Slutsky matrix.¹¹

Because (13) is rather cumbersome, we again consider the special case of quasilinear preferences to illustrate the different possibilities. Suppose preferences take the form $x_1 + F(x_2, X_3)$. Let us simplify even further by considering a marginal increase in the tax from a starting point of $t_2 = 0$. With these simplifications, equation (13) becomes

$$\frac{\partial \hat{x}_2}{\partial t_2} = \frac{-x_2 C_{23} + (1 - \pi_3)\Psi}{-C_{33}}, \quad (14)$$

and this establishes several results about how demand for x_2 will respond to imposition of a dedicated tax t_2 .

Table 1 summarizes the qualitative results, showing how they depend, in part, on the individual's marginal WTP for the public good at the initial allocation.¹² Consider the knife-edge case where the individual's marginal WTP exactly equals the per-unit cost of providing the public good (i.e., $\pi_3 = 1$). If the tax-linked goods are Hicksian complements or substitutes, then demand for the private good will increase or decrease, respectively. Notice that an increase in demand for the private good is rather counter-intuitive, because imposition of a tax increases demand. Interestingly, this suggests that sellers of good x_2 might actually benefit from imposition of the

¹¹The Slutsky matrix is equivalent to the Hessian of the expenditure function (i.e., the matrix of derivatives of the compensated demand functions), and we assume the weak inequalities implied by negative semi-definiteness hold strictly.

¹²See Banzhaf and Smith (2020) for a similar set of results, though motivated with a different setup.

Table 1: The qualitative sign of the change in demand for x_2 given imposition of a dedicated tax t_2 , assuming quasilinear preferences

Relationship between x_2 and X_3	$\pi_3 < 1$	$\pi_3 = 1$	$\pi_3 > 1$
Hicksian complements	?	+	+
Hicksian substitutes	−	−	?

dedicated tax, which is a possibility that Banzhaf and Smith (2020) explore in greater detail.

The same qualitative results occur if π_3 is greater (less) than unity, and the linked goods are complements (substitutes). As shown in the cells with question marks in Table 1, ambiguity occurs when there are effects that push in different directions, that is, when the marginal WTP is less (greater) than unity, and the linked goods are complements (substitutes). Recall, however, that we are considering the special case of quasilinear preferences here to illustrate the range of possibilities. In a more general setting, these different possibilities might occur in each of the cases. The primary insight is that imposing a dedicated tax on a private good in order to provide a public good can affect demand for the private good in what are likely to be unexpected ways.

4.2 Nash Equilibrium

We now consider a setup where all n individuals in the economy are simultaneously engaged in private provision of the public good through consumption of the private good subject to the dedicated tax. The model’s setup implies that individuals are playing a game for any given level of the dedicated tax. We therefore consider equilibrium existence and uniqueness for any given tax rate.¹³

We can write each individual’s demand for private provision as $\hat{x}_3(t_2, \tilde{X}_3) = t_2 \hat{x}_2(t_2, \tilde{X}_3)$, which is each individual’s best-response function.¹⁴ A Nash equilibrium

¹³Our examination of the public good equilibrium is one way in which our analysis differs from that in Banzhaf and Smith (2020). While they do consider an equilibrium among suppliers, we simplify the supply side of the market by assuming fixed and exogenous prices.

¹⁴Note that each individual’s demand for the *aggregate* level of the public good then follows by definition: $\hat{X}_3 = t_2 \hat{x}_2(t_2, \tilde{X}_3) + \tilde{X}_3$. Furthermore, recall that lower-case variables denote individual consumption or provision, the upper-case X_3 denotes aggregate provision, and the circumflex (“hat”) denotes the restricted demand functions.

is a fixed point at the intersection of all n best response functions. Equivalently, a Nash equilibrium is a set of choices x_2^i for all i that satisfy the first order condition (12) for all n individuals with $X_3 = t_2 \sum_{i=1}^n x_2^i$. Note that, without loss of generality, we are still normalizing the level of the public good that does not come through private provision to zero.

Kotchen (2007) establishes a sufficient condition for equilibrium existence and uniqueness in the general impure public good model, and as noted above, the setup here is a special case for a given level of t_2 . The condition is based on the slope of each individual's demand for the aggregate level of the public good with respect to the provision of others. In particular, using the notation employed here, the sufficient condition is:

$$0 < \frac{\partial \hat{X}_3(t_2, \tilde{X}_3)}{\partial \tilde{X}_3} = t_2 \frac{\partial \hat{x}_2(t_2, \tilde{X}_3)}{\partial \tilde{X}_3} + 1 \leq 1, \quad (15)$$

where $\hat{X}_3(t_2, \tilde{X}_3)$ is an individual's demand for the aggregate level of the public good, and the bridge between the two equal expressions is based on the identity $\hat{X}_3 = t_2 \hat{x}_2 + \tilde{X}_3$. Equation (15) implies that an increase in spill-ins (i.e., \tilde{X}_3) must increase demand for the public good and not decrease demand for the untaxed private good x_1 . This is essentially a normality assumption with respect to full, virtual income. A further implication is that best-response functions have slopes less than zero and greater than -1 , and this monotonicity combined with continuity ensures the existence of a unique fixed point (Kotchen 2007).

With identical individuals, a further implication is that the equilibrium will be symmetric, and we denote it simply as $x_2^N(t_2)$ for each individual, where the superscript N is used to denote a Nash equilibrium quantity. It follows that the aggregate, equilibrium level of public good provision will satisfy

$$X_3^N(t_2) = nx_3^N(t_2) = nt_2x_2^N(t_2),$$

which shows how the solution can be written in terms of each individual's level of private

provision or demand for the taxed private good.

Before turning to the optimal dedicated tax in the next section, it is helpful to establish an intermediate result here. We showed in the previous subsection that demand for x_2 can be increasing or decreasing in response to imposition of a dedicated tax t_2 . The question that we consider now is whether $\partial \hat{x}_2 / \partial t_2$ in (13) can take either sign while still satisfying the assumption in (15). The reason is that maintaining both possibilities creates some interesting results that we derive in the next section. In general, the answer is yes, which we show in Appendix A.4. This means that even imposing the constraint on individual behavior that is sufficient for a unique Nash equilibrium, it is still possible for $\partial \hat{x}_2 / \partial t_2$ to be either positive or negative.¹⁵

5 The Optimal Dedicated Tax

We now consider the optimal dedicated tax that a social planner would choose. We also compare the resulting allocation with the Pareto optimal allocation defined in Section 3. We thus compare implications of the allocation consistent with the optimal dedicated tax to that which is first-best regardless of the policy instrument.

It is important to recognize with any dedicated tax, the individuals continue to play a non-cooperative Nash game that the planner must take into account when choosing t_2 . We can write the planner's objective as:

$$\max_{t_2} nU \left(\frac{w - (p_2 + t_2)x_2^N(t_2)}{p_1}, x_2^N(t_2), nt_2x_2^N(t_2) \right), \quad (16)$$

where a key feature of this statement of the problem is that $x_2^N(t_2)$ is consistent with

¹⁵The possibility for $\partial \hat{x}_2 / \partial t_2 < 0$ is perhaps somewhat less surprising because as described earlier t_2 operates, in part, like an increase in price. But we have also discussed how it has the additional effect of increasing the level of the public good for a given level of x_2 , and the sign of the comparative static depends on the signs and relative magnitudes of multiple price and income effects. Given the condition in (15), we show in Appendix A.4 that if $\pi \leq 1$, then assuming the two goods are normal and Hicksian substitutes is sufficient for $\partial \hat{x}_2 / \partial t_2 < 0$. However, in order to obtain the opposite sign with both goods still being normal, a necessary condition is for Hicksian complements, where in particular $0 > C_{22} > C_{32}(= C_{23}) > C_{33}$. Intuitively, this corresponds to the case where x_2 has a relatively small own-price response, and it aligns with the strong (Hicksian) complements case described by Cornes and Sandler (1996, p.267), where crowding-in can occur.

the equilibrium that arises given any choice of the dedicated tax level.

The first-order condition that defines the solution can be written as

$$nU_3 \left(x_2^N + t_2 \frac{\partial x_2^N}{\partial t_2} \right) + U_2 \frac{\partial x_2^N}{\partial t_2} = \frac{U_1}{p_1} \left(x_2^N - (p_2 + t_2) \frac{\partial x_2^N}{\partial t_2} \right). \quad (17)$$

To build intuition, first consider the special case where $\frac{\partial x_2^N}{\partial t_2} = 0$, so that equation (17) simplifies to $nU_3 = \frac{U_1}{p_1}$. This expression equates the social marginal benefit of greater public good provision for an individual and the cost of foregone consumption of x_1 . More generally, however, the equilibrium quantity x_2^N will change with a change in the dedicated tax t_2 . It is also the case that the sign of $\frac{\partial x_2^N}{\partial t_2}$ can be positive or negative, in much the same way that we showed previously how the individuals demand response in equation (13) can be positive or negative.¹⁶ Equation (17) thus shows that the optimal dedicated tax is set where the social marginal benefits and costs are equated, after taking account of the net change in quantities due to (i) the direct effect of the change in t_2 , and (ii) the indirect effect on account of changes in equilibrium demand for the taxed private good x_2^N .

5.1 Efficiency

We now make comparisons between the Pareto optimal allocation defined in Section 3 and the allocation implied by the optimally set dedicated tax. In particular, the analysis is based on a comparison between the conditions specified in (8) and (12), where the latter is evaluated with a dedicated tax that solves (16). The most straightforward way to establish the key result is to begin by assuming the optimal dedicated tax does in fact implement the Pareto optimal allocation. This means that $x_i^N(t_2) = x_i^*$ for $i = 1, 2, 3$, which by definition satisfies both conditions in (8). Now, substituting the second condition of (8) into (12) and rearranging implies yields $\frac{U_3}{U_1} = \frac{1}{p_1}$, and it follows immediately that this equation can match the first condition

¹⁶See Appendix A.5 for details on the possible relationship between the signs of $\frac{\partial \hat{x}_2}{\partial t_2}$ and $\frac{\partial x_2^N}{\partial t_2}$, that is, an individual's demand response and the equilibrium demand response that accounts for the response of all other individuals.

in (8) only if $n = 1$. This means that the efficient dedicated tax can implement the Pareto optimal allocation only in the trivial case of $n = 1$. The simple intuition in this special case is that the planner can set the tax to exactly balance the individual's preferred consumption of the public good and taxed private good.

More generally, the preceding steps prove a clear result: for $n \geq 2$, the optimal dedicated tax cannot achieve the Pareto optimal allocation, which we showed could arise with lump-sum taxation. The dedicated tax is therefore a second-best policy in cases where there is an actual public good. Intuition follows from at least two observations. The first is that imposing a dedicated tax layers an additional constraint on how the planner chooses the Pareto optimal allocation, so if the constraint is binding, the dedicated tax cannot be first best. The second is that a dedicated tax does not eliminate free-riding incentives. To show this, we simply note that with a dedicated tax in place, the private marginal benefit of consuming the the taxed private good is $U_2 + t_2U_3$, whereas the greater social marginal benefit is $U_2 + nt_2U_3$, which is the same only in the special case of $n = 1$.

5.2 Level of Public Good Provision

We now turn our focus to implications for the level of public good provision. Just because the dedicated tax is, in general, a second-best policy does not tell us whether an optimally chosen dedicated tax will implement more or less of the public good, compared to the Pareto optimal allocation. Indeed, provision of the public good may in fact be the primary motive for considering a dedicated tax (in contrast to an efficiency objective).

To illustrate these results, it is useful to further simplify the condition defining the optimal dedicated tax. Substituting the equilibrium condition (12) into (17) and rearranging yields

$$\frac{U_3}{U_1} \left(n + (n - 1)\varepsilon_{x_2^N, t_2} \right) = \frac{1}{p_1}, \quad (18)$$

where $\varepsilon_{x_2^N, t_2} = \frac{t_2}{x_2^N} \frac{\partial x_2^N}{\partial t_2}$ is the elasticity representing the percentage change in each individual's *equilibrium* level of demand for the taxed private good, given a percentage

change in the tax rate. Notice that the elasticity here differs from a standard demand elasticity for two reasons. First, as mentioned previously, a change in the tax rate is not simply a change in the price because the revenue is used to provide the public good that directly enters the individual utility functions. Second, the elasticity is for an equilibrium response and not simply a demand response, meaning that the change in behavior of all others is taken into account.¹⁷ In this respect, $\varepsilon_{x_2^N, t_2}$ captures the causal impact of the tax change on behavior and is therefore related to the “policy elasticity” described by Hendren (2016).¹⁸

What, then, does the sign of $\varepsilon_{x_2^N, t_2}$ imply about the equilibrium level of public good provision compared to the Pareto optimal level? In Appendix A.6, we derive the results summarized in Table 2 under the assumption of strict concavity of the utility function in all three arguments. The table compares the relative magnitudes of the equilibrium quantities for the optimal dedicated tax with those of the Pareto optimal allocation. The signs in each cell indicate whether the dedicated-tax equilibrium quantity (of the tax private good or the public good) is greater than (+) or less than (−) the first-best quantity. The different columns correspond to different ranges of $\varepsilon_{x_2^N, t_2}$.

To build intuition for these findings, let us begin with the identity $x_3^N(t_2) = t_2 x_2^N(t_2)$. Then differentiating with respect to t_2 and rearranging, it follows that

$$\text{sign} \left(\frac{\partial x_3^N}{\partial t_2} \right) = \text{sign} \left(1 + \varepsilon_{x_2^N, t_2} \right).$$

This means, for example, that a marginal increase in the dedicated tax will increase (decrease) the equilibrium level of the public good depending on whether $\varepsilon_{x_2^N, t_2}$ is greater (less) than -1 .¹⁹ When the elasticity is less than -1 , we see in Table 2 that

¹⁷In Appendix A.5, we show the exact relationship between the equilibrium elasticity $\varepsilon_{x_2^N, t_2}$ and the demand response elasticity, denoted $\varepsilon_{\hat{x}_2, t_2}$. In general, we find nothing that rules out the possibility for the equilibrium elasticity to take either sign, and in most cases, it will have the same sign as the demand response elasticity.

¹⁸The linkages between our analysis and that of Hendren (2016) would be an interesting avenue for further investigation.

¹⁹Note that satisfying (18) means that for an interior solution, it must hold that the elasticity satisfies $\varepsilon_{x_2^N, t_2} > -n/(n-1)$.

Table 2: Difference in quantities between the optimal dedicated tax equilibrium and the Pareto optimal allocation for $n \geq 2$

	$\varepsilon_{x_2^N, t_2} \leq -1$	$\varepsilon_{x_2^N, t_2} \in (-1, 0)$	$\varepsilon_{x_2^N, t_2} \geq 0$
$X_3^N - X_3^*$	-	?	+
$x_2^N - x_2^*$	+	?	-

the dedicated tax results is a relatively lower level of the public good. This is intuitive because increasing the tax further would only serve to further lower the level of the public good. When the elasticity is greater than zero, the equilibrium level of the public good is greater with the dedicated tax, and this is driven by the way that increasing the tax has the additional effect of increasing demand for the taxed good. Finally, in the intermediate inelastic case, where $-1 < \varepsilon_{x_2^N, t_2} < 0$, either result is possible, as increasing the tax decreases demand for the taxed private good and yet the quantity of the public good still increases.

5.3 Summary

We have illustrated two main results in this section. The first is that an optimally chosen dedicated tax will not, in general, implement the Pareto optimal allocation. The second is that, despite it being a second-best policy, the optimally chosen dedicated tax can implement an equilibrium level of the public good that exceeds or falls short of the first-best level, depending on the sign and magnitude of the equilibrium tax elasticity of demand for the taxed private good.

6 Generalization with Direct Donations

We have thus far assumed the only way that individuals can provide the public good is through consumption of the taxed private good. In this section, we generalize the model's setup to allow the possibility for individuals to make a direct contribution to the public good, in addition to the possibility of provision through the taxed private good. The expanded choice set, with multiple channels for public good provision, is

similar to that in Kotchen (2005, 2006) and Chan and Kotchen (2014). An example setting where the setup applies is a community where there exists a real-estate transfer tax that is used to fund the acquisition of conservation lands, while at the same time a land trust is operating with the same objective. This means that individuals are subject to the tax, which provides the public good, while also having the opportunity to make voluntary contributions to the land trust. Here we consider how this might change the results.

Prior to implementation of a dedicated tax, we can write the individual's problem as

$$\max_{x_1, x_2, x_3} U(x_1, x_2, x_3 + \tilde{X}_3) \text{ s.t. } w = p_1 x_1 + p_2 x_2 + x_3,$$

where the difference here is that the individual can choose x_3 directly. This is the standard setup for private provision of a public good (Bergstrom et al. 1986). Substituting the budget constraint into the maximand and choosing x_2 and x_3 , the Kuhn-Tucker first-order conditions are

$$\begin{aligned} x_2 &\geq 0, & -\frac{p_2}{p_1}U_1 + U_2 &\leq 0, & c.s. \\ x_3 &\geq 0, & -\frac{1}{p_1}U_1 + U_3 &\leq 0, & c.s. \end{aligned} \tag{19}$$

where *c.s.* denotes the complementary slackness condition. Note that the Pareto optimal allocation with this setup remains the same as defined in Section 3.

An equivalent and useful way to write the individual's problem is with the implicit choice of the aggregate level of public good provision:

$$\max_{x_1, x_2, X_3} U(x_1, x_2, X_3) \text{ s.t. } w + \tilde{X}_3 = p_1 x_1 + p_2 x_2 + X_3 \text{ and } X_3 \geq \tilde{X}_3,$$

where the corresponding first-order conditions are identical to those above. Following convention for the pure public model (see Bergstrom et al. 1986), we can write the solution as

$$X_3 = f(w + \tilde{X}_3),$$

where we have suppressed notation for prices. The argument in $f(\cdot)$ is the individual's

full income: wealth plus the value of public good spill-ins. The standard assumption is to assume normality of all goods, in which case $0 < f' \leq 1$, where the inequality holds strictly for an interior solution. This guarantees equilibrium existence and uniqueness. This follows because best-response functions $x_3 = f(w + \tilde{X}_3) - \tilde{X}_3$ are continuous and non-increasing, which gives a unique fixed point for each individual's direct contribution (reasons identical to those described in Section 4.2).

The first-order condition establishing the trade-off between x_1 and x_2 , along with the budget constraint, then defines each individual's choice of the private goods. The equilibrium level of the public good will therefore satisfy

$$\bar{X}_3 = f(w + (n - 1)\bar{x}_3),$$

where the overbar represents the equilibrium quantity in this new setting with direct donations. Moreover, each individual's equilibrium choice is denoted $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ and it follows by definition and symmetry that $\bar{x}_3 = \bar{X}_3/n$.

We now introduce the dedicated tax to this more general setup. The individual's problem is

$$\max_{x_1, x_2, x_3} U(x_1, x_2, t_2 x_2 + x_3 + \tilde{X}_3) \text{ s.t. } w = p_1 x_1 + (p_2 + t_2) x_2 + x_3,$$

which shows the two ways to potentially provide the public good: through consumption of x_2 , through a direct donation x_3 , or both. In this case, the Kuhn-Tucker first-order conditions are

$$\begin{aligned} x_2 \geq 0, \quad & -\frac{(p_2+t_2)}{p_1}U_1 + U_2 + t_2U_3 \leq 0, \quad c.s. \\ x_3 \geq 0, \quad & -\frac{1}{p_1}U_1 + U_3 \leq 0, \quad c.s. \end{aligned} \tag{20}$$

Notice that only the first condition in (20) differs from that in (19). Assuming there is an interior solution, we can substitute the second condition in (20) into the first and find that the conditions defining an equilibrium are identical to those in (19). This establishes a neutrality result: implementing a dedicated tax has no effect on

the equilibrium if individuals continue to purchase the private goods and make a direct donation after the tax is imposed. The intuition is that provision through the dedicated tax crowds out direct donations one-for-one.²⁰

To see the crowding out of one's own direct donations more directly, consider how the equilibrium level of the public good at an interior solution will satisfy

$$\bar{X}_3 = n(t_2\bar{x}_2 + \bar{x}_3).$$

Totally differentiating, and recognizing that $d\bar{X}_3 = 0$ because of the same conditions in (19) and (20), we have

$$\frac{d\bar{x}_3}{dt_2} = -\bar{x}_2(1 + \varepsilon_{\bar{x}_2, t_2}).$$

Yet because the equilibrium allocation of all three goods does not change, it must also hold that $\varepsilon_{\bar{x}_2, t_2} = 0$, meaning that $\frac{d\bar{x}_3}{dt_2} = -\bar{x}_2$. This illustrates how direct donations are decreasing in the dedicated tax rate such that the crowding out is one-for-one. This follows because, given a change in t_2 , the change in an individual's contribution to the public good through consumption of the taxed private good is exactly \bar{x}_2 .²¹

A further implication is that we can define the threshold value $t'_2 = \bar{X}_3/n\bar{x}_2$, which is the level of the tax that exactly reduces direct donations to zero and maintains the same equilibrium allocation. We have thus shown complete neutrality for all $t_2 \leq t'_2$. Things change, however, for dedicated tax levels greater than t'_2 , where an additional increase in the tax rate will render direct donations irrelevant as they are completely

²⁰Kotchen (2006) shows that group size must be sufficiently small for interior solutions in the presence of a impure public good and the opportunity to make a direct donation. The same result applies with the setup here, which means that the neutrality result depends on a sufficiently small n . With large n , direct donations will be fully crowded out and the model reverts back that considered in the previous sections. We believe that analyzing how properties of dedicated taxes with public goods depend on group size is a question worthy of additional research, drawing on some of the results found in both Andreoni (1988) and Kotchen (2006).

²¹A well-known result in the charitable giving literature is that one-for-one crowding out no longer occurs in the model of impure altruism (Andreoni 1989, 1990). In the setting considered here, however, the crowding effects continue to hold exactly if warm-glow benefits are derived equally from the public good provision that occurs through either direct donations or the dedicated tax. This follows because both alternatives are perfect substitutes with respect to public good provision at interior solutions. Nevertheless, the perfect substitutability will no longer hold if only one form of giving (e.g., direct donations) confers warm glow benefits while the other (e.g., indirect donations via tax payments) does not. In such cases, the crowding effect will be mitigated in ways consistent with that underlying the standard results in the literature.

crowded out by tax revenues. Thus, for $t_2 > t'_2$, the model with direct donations reduces to the simpler setup in previous sections.

7 Conclusion

This paper set out to examine some of the advantages and disadvantages of using dedicated taxes to finance the provision of public goods. We show how the impure public good model provides a useful way for understanding the positive and normative consequences of dedicated taxes. We began by showing how imposition of a dedicated tax can, somewhat counter-intuitively, increase or decrease demand for the taxed good. We then showed how the optimal dedicated tax cannot in general achieve the Pareto optimal allocation. It can, however, generate a conditionally efficient equilibrium with comparatively more or less of the public good, depending, in part, on whether the public good and the taxed private good are Hicksian complements or substitutes, respectively. We also show a neutrality result: when individuals have the opportunity to make direct donations, dedicated taxes that are sufficiently low will have no effect on the equilibrium allocation.

Several of these results may help to illuminate the potential political economy and policy implications of dedicated taxes. For example, the possibility that dedicated taxes can stimulate demand for the taxed good suggests suppliers may actually support dedicated taxes (see also Banzhaf and Smith 2020). Indeed, imposing dedicated taxes may provide a mechanism with benefits akin to corporate social responsibility that links public and private goods (Besley and Ghatak 2007), where all producers must meet the same standards rather than having it be met voluntarily. A similar mechanism, albeit voluntary in membership, is the 1% For the Planet initiative (mentioned previously), where a group of businesses, including companies like Patagonia, donate 1% of gross sales to environmental non-profit organizations. In this case, the dedicated tax is effectively 1% of sales.

The neutrality result also highlights one of the potential risks of dedicated taxes: the crowding out of direct donations. Consider, for example, how federal funding for

wildlife and habitat conservation is often disbursed contingent upon matching funds from states, as is the case with the Pittman-Robertson and Dingell-Johnson Acts. While states typically raise matching funds from hunting and fishing licenses, they also rely on contributions from private organizations, with Ducks Unlimited being a frequent and significant partner with state agencies for securing federal money. Our findings point to the potential unintended crowding out that may accompany a state's greater reliance on dedicated taxes. If contributions from organization such as Ducks Unlimited are crowded out, then the dedicated taxes may have limited or no effect on total conservation funding. Along these lines, Walls and Ashenfarb (2020) also warns of a second dimension of crowding out, whereby dedicated taxes may crowd out allocations from general revenues.

A Appendix

A.1 Derivation of Equation (6)

We use two equations to solve for changes in the virtual magnitudes: the definition of virtual full income

$$\mu(p_1, p_2, w, \tilde{X}_3) = w + \pi_3(p_1, p_2, w, \tilde{X}_3)\tilde{X}_3$$

and the constraint on the level of the public good

$$X_3(p_1, p_2, \pi_3(\bullet), \mu(\bullet)) = \tilde{X}_3.$$

Differentiating both of these with respect to \tilde{X}_3 yields

$$\frac{\partial \mu}{\partial \tilde{X}_3} = \frac{\partial \pi_3}{\partial \tilde{X}_3} \tilde{X}_3 + \pi_3,$$

and

$$X_{33} \frac{\partial \pi_3}{\partial \tilde{X}_3} + X_{3\mu} \frac{\partial \mu}{\partial \tilde{X}_3} = 1.$$

We can obtain the following matrix representation for this system of equations:

$$\begin{bmatrix} X_{33} & X_{3\mu} \\ \tilde{X}_3 & -1 \end{bmatrix} \begin{bmatrix} \frac{\partial \pi_3}{\partial \tilde{X}_3} \\ \frac{\partial \mu}{\partial \tilde{X}_3} \end{bmatrix} = \begin{bmatrix} 1 \\ -\pi_3 \end{bmatrix}.$$

Using Cramer's rule, we can solve for

$$\begin{aligned}\frac{\partial \pi_3}{\partial \tilde{X}_3} &= \frac{\det \begin{bmatrix} 1 & X_{3\mu} \\ -\pi_3 & -1 \end{bmatrix}}{\det \begin{bmatrix} X_{33} & X_{3\mu} \\ \tilde{X}_3 & -1 \end{bmatrix}} = \frac{-1 + \pi_3 X_{3\mu}}{-X_{33} - X_3 X_{3\mu}} \\ \frac{\partial \mu}{\partial \tilde{X}_3} &= \frac{\det \begin{bmatrix} X_{33} & 1 \\ \tilde{X}_3 & -\pi_3 \end{bmatrix}}{\det \begin{bmatrix} X_{33} & X_{3\mu} \\ \tilde{X}_3 & -1 \end{bmatrix}} = \frac{-X_{33}\pi_3 - \tilde{X}_3}{-X_{33} - X_3 X_{3\mu}} = \frac{X_{33}\pi_3 + \tilde{X}_3}{X_{33} + X_3 X_{3\mu}}.\end{aligned}$$

We can further simplify these expressions by substituting in the Slutsky decomposition, $X_{33} = C_{33} - X_3 X_{3\mu}$, to obtain

$$\begin{aligned}\frac{\partial \pi_3}{\partial \tilde{X}_3} &= \frac{-1 + \pi_3 X_{3\mu}}{-X_{33} - X_3 X_{3\mu}} \\ &= \frac{-1 + \pi_3 X_{3\mu}}{-C_{33}} \\ &= \frac{1 - \pi_3 X_{3\mu}}{C_{33}} \\ \frac{\partial \mu}{\partial \tilde{X}_3} &= \frac{X_{33}\pi_3 + \tilde{X}_3}{X_{33} + X_3 X_{3\mu}} \\ &= \frac{(C_{33} - X_3 X_{3\mu})\pi_3 + \tilde{X}_3}{C_{33}} \\ &= \frac{X_3 X_{3\mu}\pi_3 - \tilde{X}_3}{-C_{33}} + \pi_3 \\ &= \left[\frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right] \tilde{X}_3 + \pi_3.\end{aligned}$$

Note that the last step uses the constraint $X_3 = \tilde{X}_3$. Substituting these expressions into (5) and rearranging immediately yields (6).

To show this, we have from (5) that

$$\begin{aligned}
\frac{\partial \hat{x}_j(p_1, p_2, w, \tilde{X}_3)}{\partial \tilde{X}_3} &= [C_{j3} - X_3 x_{j\mu}] \frac{\partial \pi_3}{\partial \tilde{X}_3} + x_{j\mu} \frac{\partial \mu}{\partial \tilde{X}_3} \\
&= [C_{j3} - X_3 x_{j\mu}] \left(\frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right) + x_{j\mu} \left(\left[\frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right] \tilde{X}_3 + \pi_3 \right) \\
&= C_{j3} \left(\frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right) - X_3 x_{j\mu} \left(\frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right) + x_{j\mu} \left[\frac{1 - \pi_3 X_{3\mu}}{C_{33}} \right] \tilde{X}_3 + \pi_3 x_{j\mu}.
\end{aligned}$$

Using the constraint $X_3 = \tilde{X}_3$, we can cancel the middle terms to obtain (6)

$$\frac{\partial \hat{x}_j(p_1, p_2, w, \tilde{X}_3)}{\partial \tilde{X}_3} = \frac{C_{j3}(1 - \pi_3 X_{3\mu})}{C_{33}} + \pi_3 x_{j\mu}.$$

A.2 Derivation of Conditions (7) and (8)

Allowing heterogeneous preferences, endowments, and arbitrary welfare weights a_i for all i , the set of Pareto optimal allocations must solve

$$\max_{\{x_1\}^i, \{x_2\}^i} \sum_{i=1}^n a_i U^i(x_1^i, x_2^i, Y) \text{ s.t. } Y = \sum_{i=1}^n (w^i - p_1 x_1^i - p_2 x_2^i).$$

The first-order conditions are

$$\begin{aligned}
\sum_{i=1}^n a_i p_1 U_3^i &= a_i U_1^i \text{ for all } i \\
\sum_{i=1}^n a_i p_2 U_3^i &= a_i U_2^i \text{ for all } i.
\end{aligned}$$

The first implies that $a_i U_1^i$ is equal for all i , and the second implies that $a_i U_2^i$ is equal for all i . With a bit of rearranging, these first-order conditions immediately imply the conditions in (7). Assuming further that all individuals have identical preferences and that $a_i = a$ for all i , the first-order conditions above further imply the conditions in (8), which define the symmetric, Pareto optimal allocation.

A.3 Derivation of Equation (13)

The solution to (11) will satisfy the unrestricted solution based on virtual magnitudes such that

$$\hat{x}_2(t_2, \tilde{X}_3) = x_2(\pi_2(\cdot), \pi_3(\cdot), \mu(\cdot)), \quad (21)$$

where the virtual magnitudes are themselves functions of the exogenous parameters $(p_1, p_2, w, \tilde{X}_3, t_2)$. π_2 is defined analogously to π_3 in the main text; it is the virtual price of x_2 , and it newly appears because the dedicated tax leads to joint production of x_2 and x_3 . It will, however, cancel out in the steps below. Differentiating equation (21), the comparative static of interest can be expressed generally as

$$\frac{\partial \hat{x}_2}{\partial t_2} = x_{22} \frac{\partial \pi_2}{\partial t_2} + x_{23} \frac{\partial \pi_3}{\partial t_2} + x_{2\mu} \frac{\partial \mu}{\partial t_2}. \quad (22)$$

Following the method outlined in of Cornes and Sandler (1996, p.262,295), the three equations that the virtual magnitudes must satisfy are based on the tax-inclusive price of x_2 equaling the value of what it buys in terms of virtual magnitudes

$$p_2 + t_2 = \pi_2(\cdot) + t_2 \pi_3(\cdot),$$

the quantity relationship between x_2 and the public good

$$t_2 x_2(\pi_2(\cdot), \pi_3(\cdot), \mu(\cdot)) = X_3(\pi_2(\cdot), \pi_3(\cdot), \mu(\cdot)) - \tilde{X}_3,$$

and the definition of virtual full income

$$\mu(\cdot) = w + \pi_3(\cdot) \tilde{X}_3.$$

Differentiating these equations with respect to t_2 , using Cramer's rule and the Slutsky decomposition, we can solve for changes in each of the virtual magnitudes:

$$\begin{aligned}\frac{\partial \pi_2}{\partial t_2} &= \frac{t_2 x_2 - (1 - \pi_3)(C_{33} - t_2 C_{23}) - (1 - \pi_3)(t_2 x_2)(t_2 x_{2\mu} - X_{3\mu})}{\Omega}, \\ \frac{\partial \pi_3}{\partial t_2} &= \frac{-x_2 + (1 - \pi_3)(C_{32} - t_2 C_{22}) + (1 - \pi_3)x_2(t_2 x_{2\mu} - X_{3\mu})}{\Omega}, \\ \frac{\partial \mu}{\partial t_2} &= \frac{\partial \pi_3}{\partial t_2} \tilde{X}_3.\end{aligned}$$

Substituting these three expressions into (22), using the Slutsky decomposition again, and rearranging immediately yields equation (13).

A.4 Verification that $\partial \hat{x}_2 / \partial t_2 \leq 0$ Given Assumption (15)

We have already established that

$$\frac{\partial \hat{x}_2}{\partial t_2} = \frac{x_2(t_2 C_{22} - C_{23}) + (1 - \pi_3)\Psi + (1 - \pi_3)x_2[(C_{33} - t_2 C_{32})x_{2\mu} + (t_2 C_{22} - C_{23})X_{3\mu}]}{\Omega}.$$

Following the same steps for deriving a comparative static result using virtual magnitudes, it can be shown that

$$\frac{\partial \hat{X}_3}{\partial \tilde{X}_3} = \frac{(t_2 C_{32} - C_{33}) + t_2 \pi_3 [(C_{23} - t_2 C_{22})X_{3\mu} + (t_2 C_{32} - C_{33})x_{2\mu}]}{\Omega},$$

and assumption (15) requires that $0 < \partial \hat{X}_3 / \partial \tilde{X}_3 \leq 1$. To simplify notation, and without loss of generality, we normalize $t_2 = 1$ and let $A \equiv C_{32} - C_{33}$ and $B \equiv C_{23} - C_{22}$. Note that $A + B = \Omega > 0$, and recall that $\Psi < 0$. The two equations above simplify to

$$\frac{\partial \hat{x}_2}{\partial t_2} = \frac{-x_2 B + (1 - \pi_3)\Psi + (1 - \pi_3)x_2[-Ax_{2\mu} - BX_{3\mu}]}{A + B} \quad (23)$$

and

$$\frac{\partial \hat{X}_3}{\partial \tilde{X}_3} = \frac{A + \pi_3[Ax_{2\mu} + BX_{3\mu}]}{A + B}. \quad (24)$$

Using the notation in (24), satisfying the conditions in (15) requires both of the following:

$$A + \pi_3 [Ax_{2\mu} + BX_{3\mu}] > 0 \quad (25)$$

and

$$\pi_3 [Ax_{2\mu} + BX_{3\mu}] - B \leq 0. \quad (26)$$

With a bit of rearranging to (23), it is straightforward to verify that

$$\text{sign} \left(\frac{\partial \hat{x}_2}{\partial t_2} \right) = \text{sign} \left(\pi_3 [Ax_{2\mu} + BX_{3\mu}] - B + (1 - \pi_3) \frac{\Psi}{x_2} - [Ax_{2\mu} + BX_{3\mu}] \right).$$

Because our objective is to show different possibilities, we can get some traction by assuming $\Psi \rightarrow 0$, as nothing rules out this possibility. In this case, we have

$$\text{sign} \left(\frac{\partial \hat{x}_2}{\partial t_2} \right) = \text{sign} \left(\underbrace{\pi_3 [Ax_{2\mu} + BX_{3\mu}] - B}_{\leq 0} - [Ax_{2\mu} + BX_{3\mu}] \right), \quad (27)$$

where the inequality follows from (26) This expression shows how the sign will depend primarily on the sign and magnitudes of price and income effects.

To focus on what is perhaps the most plausible scenario, let us assume that both x_2 and X_3 are normal (i.e., $x_{2\mu}, X_{3\mu} \geq 0$). In this case, it follows that if the two goods are Hicksian substitutes (i.e., $A, B > 0$), the sign of (27) is always negative. It only remains to show the possibility for (27) to be positive without violating any of the conditions. Clearly, a necessary condition is $Ax_{2\mu} + BX_{3\mu} < 0$. Continuing to assume both goods are normal, this requires either A or B negative. But (25) implies that we cannot satisfy the inequality and have $A < 0$. Hence $A > 0$ and $B < 0$ is a necessary condition, and this means that the two goods are Hicksian complements in the case where $0 > C_{22} > C_{32}(= C_{23}) > C_{33}$. Given that this is plausible, we therefore find nothing that rules out the possibility for (27) to be positive.

A.5 The relationship between the sign of $\varepsilon_{x_2^N, t_2}$ and $\varepsilon_{\hat{x}_2, t_2}$

We begin with the identity

$$x_2^N(t_2) = \hat{x}_2(t_2, (n-1)t_2 x_2^N(t_2)).$$

Differentiating yields

$$\frac{\partial x_2^N}{\partial t_2} = \frac{\partial \hat{x}_2}{\partial t_2} + \frac{\partial \hat{x}_2}{\partial \tilde{X}_3} (n-1) \left(x_2^N + t_2 \frac{\partial x_2^N}{\partial t_2} \right),$$

and rewriting with elasticities implies

$$\varepsilon_{x_2^N, t_2} = \frac{\varepsilon_{\hat{x}_2, t_2} + D}{1 - D}, \quad (28)$$

where $D \equiv (n-1) \frac{\partial \hat{x}_3}{\partial \tilde{X}_3} \leq 0$, and the sign follows by the definition $\frac{\partial \hat{x}_3}{\partial \tilde{X}_3} = \frac{\partial \hat{X}_3}{\partial \tilde{X}_3} - 1$ combined with the assumption in (15). Equation (28) indicates that $\varepsilon_{x_2^N, t_2}$ shares the same sign as $\varepsilon_{\hat{x}_2, t_2}$ in all cases except for one, where $\varepsilon_{\hat{x}_2, t_2} > 0$ and the numerator of (28) is negative, which is more likely to occur with large n , greater crowding out, or both. More generally, we find that nothing rules out the possibility for $\varepsilon_{x_2^N, t_2}$ to take either sign.

A.6 Derivation of the Relative Magnitudes in Table 2

Let us assume $n \geq 2$. The conditions that define the Pareto optimal allocation in (8) can be written as follows:

$$\begin{aligned} \frac{U_3^*}{U_1^*} &= \frac{1}{p_1 n} \\ \frac{U_2^*}{U_1^*} &= \frac{p_2}{p_1}. \end{aligned}$$

The conditions that define the allocation consistent with the optimal dedicated tax in (18) and (12) can be written in parallel fashion as

$$\begin{aligned}\frac{U_3^N}{U_1^N} &= \frac{1}{p_1(n + (n-1)\varepsilon_{x_2^N, t_2})} \\ \frac{U_2^N}{U_1^N} &= \frac{p_2 + t_2 \left(1 - \frac{1}{n+(n-1)\varepsilon_{x_2^N, t_2}}\right)}{p_1}\end{aligned}$$

where the second comes from substituting the first into (12) and rearranging. Comparing these conditions, it follows that

$$\frac{U_3^N}{U_1^N} \geq \frac{U_3^*}{U_1^*} \iff \varepsilon_{x_2^N, t_2} \leq 0$$

and

$$\frac{U_2^N}{U_1^N} \geq \frac{U_2^*}{U_1^*} \iff \varepsilon_{x_2^N, t_2} \geq -1.$$

Three cases are then useful to consider:

$$\varepsilon_{x_2^N, t_2} \geq 0 \implies \frac{U_3^*}{U_1^*} \geq \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} < \frac{U_2^N}{U_1^N}$$

$$\varepsilon_{x_2^N, t_2} \in (-1, 0) \implies \frac{U_3^*}{U_1^*} < \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} < \frac{U_2^N}{U_1^N}.$$

$$\varepsilon_{x_2^N, t_2} \leq -1 \implies \frac{U_3^*}{U_1^*} < \frac{U_3^N}{U_1^N} \text{ and } \frac{U_2^*}{U_1^*} \geq \frac{U_2^N}{U_1^N}.$$

The results in Table 2 follow by aligning each of these conditions with strict concavity with respect to each argument of the utility function, along with use of the budget constraint.

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