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A comment on the paper by Lee (1998): "On micrometeorological observations of surface-air exchange over tall vegetation"

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Abstract

We examine the analysis by Lee (1998) of the scalar conservation budget in advective flows. Lee treated the budget in a one-dimensional framework, neglected horizontal derivatives of turbulent quantities and proposed that mean flow advection, simplified to its vertical component, can be used to improve budget closure, when data from only a single tower are available. We conclude:

- that the appropriate analysis framework for constructing such budgets is unavoidably two- or three-dimensional because 2D and 3D mean velocity fields always induce streamwise variation in the eddy fluxes of scalars.
- that in such flow fields it is generally incorrect to assume that the vertical component of advection $\bar{w}\partial\bar{c}/\partial z$ is everywhere much larger than the horizontal component $\bar{u}\partial\bar{c}/\partial x$ The vertical component can only provide a good measure of total advective flux divergence in the special circumstance where the tower is located beneath the vertical stagnation streamline of a recirculating flow. By referring to a linear model of scalar transport over a hill we show that the relationship between $\bar{u}\partial\bar{c}/\partial x$ and $\bar{w}\partial\bar{c}/\partial z$ is entirely dependent on particular flow conditions and that, in general, $\bar{w}\partial\bar{c}/\partial z$ cannot even be used to provide a bound on the magnitude of total advection.
- that for measurements at heights small compared to the horizontal scale of the advective flow, the horizontal gradient of turbulent flux $\partial\overline{u'c'}/\partial x$ can probably be neglected relative to its vertical equivalent $\partial\overline{w'c'}/\partial z$ and,
- by using simple hydrodynamic models of 2D flows it can be shown that the vertical gradient of mean vertical velocity is approximately constant over tower heights small compared to the horizontal scale of the advective flow.

We also comment on the proper choice of coordinate frame for analysis.

Keywords: Advection; Atmospheric boundary layer; Canopy; Carbon budget; Surface energy balance; Turbulent transport

1. Introduction

Lee (1998) has produced a timely reevaluation of the way that the mass conservation equation is used to deduce net ecosystem exchanges (NEE) of carbon and energy between the biosphere and atmosphere, given measurements of the appropriate eddy flux (e.g. Goulden et al., 1996). In this note we will argue that, while Lee's approach goes some way towards improving the current practice in particular circumstances, it is not generally applicable because it

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ignores the essential two- or three-dimensionality of advective flows.

Lee considers the situation where two- or three-dimensional flow fields are superimposed on a one-dimensional, horizontally homogeneous background flow. The inhomogeneous flows he considers to be generated by the passage of the cells of the convective, planetary boundary layer, by 'mesoscale circulations'-convective cells driven by local contrasts in the surface energy balance and, therefore, fixed in the landscape-and especially by anabatic or katabatic flows caused by the interplay of topography and diabatic forcing.

In circumstances where the measuring height and the scale of vertical variation in mean quantities is much smaller than the horizontal scale of the advective flow, it is reasonable to expect that $\partial\bar{\phi}/\partial x \ll \partial\bar{\phi}/\partial z$ where $\bar{\phi}$ is an arbitrary mean quantity and it is on these grounds that Lee has argued that in such flows the horizontal advection term $\bar{u}\partial\bar{c}/\partial x$ may be neglected relative to the vertical advection term $\bar{w}\partial\bar{c}/\partial z$. Unfortunately, exactly the same scaling arguments suggest that in these situations $\bar{w} \ll \bar{u}$ to the degree that $\bar{u}\partial\bar{\phi}/\partial x \cong \bar{w}\partial\bar{\phi}/\partial z$ and the question of whether one or the other term may be neglected in particular circumstances cannot be resolved without considering flow dynamics. In this note simple dynamic models of relevant flows will be used to test this and other assumptions in Lee (1998).

2. Theory

The conservation of a scalar c is governed by:

$$\left(\frac{\partial c}{\partial t}\right) + \left(\frac{\partial uc}{\partial x}\right) + \left(\frac{\partial wc}{\partial z}\right) = s(x, z, t) \tag{2.1}$$

where x is aligned with the local mean wind direction (assumed to be invariant with z in the measurement domain), z is perpendicular to the local terrain surface and u, w are velocity components parallel to x and z, respectively. $s(x, z)$ is the specific source strength of c and molecular diffusion has been ignored. For simplicity, throughout this note we will restrict ourselves to 2D flows. The extension to 3D is obvious and adds nothing essential to the discussion.

After Reynolds decomposition and averaging we obtain:

$$\frac{\partial\bar{c}}{\partial t} + \bar{u}\frac{\partial\bar{c}}{\partial x} + \bar{w}\frac{\partial\bar{c}}{\partial z} + \bar{c}\left(\frac{\partial\bar{u}}{\partial x} + \frac{\partial\bar{w}}{\partial z}\right) + \frac{\partial\overline{u'c'}}{\partial x} + \frac{\partial\overline{w'c'}}{\partial z} = \bar{s} \tag{2.2}$$

where overbars denote ensemble mean quantities and primes departures therefrom. The grouping of terms in Eq. (2.2) is somewhat more natural than in Lee's equation (XL2) as the fourth term on the LHS of (2.2) is identically zero by continuity. (prefix XL will denote equation numbers in Lee (1998)).

We wish to obtain the net ecosystem exchange, NEE, that is the flux of carbon or energy across the boundary between the atmosphere and the soil and vegetation. We make this the lower boundary of a control volume whose upper and lateral boundaries are located in the air. Writing Eq. (2.2) in tensor form, we can express mass conservation in this control volume as

$$\int_{-L_x}^{+L_x}\int_{-L_y}^{+L_y}\int_{0}^{z_r}\left\{\frac{\partial\bar{c}}{\partial t} + \frac{\partial\bar{u}_j\bar{c}}{\partial x_j} + \frac{\partial\overline{u'_j c'}}{\partial x_j}\right\}\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \int_{-L_x}^{+L_x}\int_{-L_y}^{+L_y}\int_{0}^{z_r}\bar{s}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{2.3}$$

where the horizontal extent of the control volume is a rectangle of side $4L_xL_y$ and its height is z_r. We can stipulate that the control volume be of unit width in the y direction ($L_y = 1/2$) and integrate (2.2) according to (2.3) to obtain:

$$\frac{1}{2L}\int_{-L}^{L}\mathrm{d}x\int_{0}^{z_r}\mathrm{d}z\left\{\frac{\partial\bar{c}}{\partial t} + \bar{u}\frac{\partial\bar{c}}{\partial x} + \bar{w}\frac{\partial\bar{c}}{\partial z} + \frac{\partial\overline{u'c'}}{\partial x}\right\} + \frac{1}{2L}\int_{-L}^{L}\overline{w'c'}(x, z_r)\,\mathrm{d}x = \mathrm{NEE} = \frac{1}{2L}\int_{-L}^{L}\mathrm{d}x\left\{\int_{0}^{z_r}\bar{s}(x,z)\,\mathrm{d}z + \overline{w'c'}(x,0)\right\} \tag{2.4}$$

where $L = L_x$ and as well as integrating dx we have divided by the streamwise extent of the control volume so we are considering average values of all quantities that vary with x. We have also written $\overline{w'c'}(0)$ as shorthand for the surface flux of c, which must of course be effected by molecular diffusion at the surface, and we have ignored complications caused by the spatial variation of mean quantities in the canopy, which requires us to apply a volume as well as a time average in order to obtain meaningful within-canopy variables. These technical details can be followed in various sources, e.g., Kaimal and Finnigan (1994) and do not affect the current arguments.

Eq. (2.4) is the appropriate framework in which to discuss mass conservation when the flowfield is two-dimensional. In his discussion of (2.4) Lee makes four assumptions:

- a) That the mass conservation equation can be treated as if it were one-dimensional even though the underlying flow field is two-dimensional, that is, values of variables at ($x = 0$, z) equal their horizontal averages, viz.

$$\frac{1}{2L}\int_{-L}^{L}\phi(x,z)\,\mathrm{d}x \cong \phi(0,z) \tag{2.5a}$$

where $\phi(x, z)$ is an arbitrary variable.

- b) $$\left(\bar{u}\frac{\partial\bar{c}}{\partial x}\right) \ll \left(\bar{w}\frac{\partial\bar{c}}{\partial z}\right) \tag{2.5b}$$

everywhere in the flow,

- c) $$\left(\frac{\partial\overline{u'c'}}{\partial x}\right) \ll \left(\frac{\partial\overline{w'c'}}{\partial z}\right) \tag{2.5c}$$

everywhere in the flow, and

- d) $$\frac{\partial\bar{w}}{\partial z} = \frac{\bar{w}_r}{z_r} \tag{2.5d}$$

We will first follow Lee and develop (2.4) using these assumptions and then go back and examine them critically. Using assumptions (2.5 a,b,c), (2.4) becomes:

$$\int_0^{z_r}\mathrm{d}z\left\{\frac{\partial\bar{c}}{\partial t} + \bar{w}\frac{\partial\bar{c}}{\partial z}\right\} + \overline{w'c'}(z_r) = \mathrm{NEE}$$
$$= \int_0^{z_r}\bar{s}(z)\,\mathrm{d}z + \overline{w'c'}(x,0) \tag{2.6}$$

And using assumption (2.5d) we obtain:

$$\mathrm{NEE} = \int_0^{z_r}\bar{s}\,\mathrm{d}z + (\overline{w'c'})_{z=0}$$
$$= \int_0^{z_r}\frac{\partial\bar{c}}{\partial t}\mathrm{d}z + (\overline{w'c'})_{z=z_r} + \bar{w}_r(\bar{c}_r - \langle\bar{c}\rangle) \tag{2.7}$$

where,

$$\langle\bar{c}\rangle = \frac{1}{z_r}\int_0^{z_r}\bar{c}(z)\,\mathrm{d}z$$

The last term in (2.7) is the result of the following integration by parts:

$$\int_0^{z_r}\bar{w}\frac{\partial\bar{c}}{\partial z}\,\mathrm{d}z = [\bar{w}\bar{c}]_0^{z_r} - \int_0^{z_r}\frac{\partial\bar{w}}{\partial z}\bar{c}\,\mathrm{d}z = [\bar{w}\bar{c}]_0^{z_r} - \frac{\bar{w}_r}{z_r}\int_0^{z_r}\bar{c}\,\mathrm{d}z$$
$$= \bar{w}_r(\bar{c}(z_r) - \langle\bar{c}\rangle) \tag{2.8}$$

and so relies on assumption (2.5d) in an essential way.

Let us now examine in detail the four assumptions (2.5) that led from (2.4) to (2.7).

2.1. Assumption (2.5a)

Imagine we have steady 1D flow in which a constant source or sink $\bar{s}(x, 0)$ at the surface together with a 1D turbulent flow generates a flux of c with a concentration profile $\bar{c}(z)$. We can abandon the complication of a canopy for this simple example. The situation is described by:

$$\bar{u}_j\frac{\partial\bar{c}}{\partial x_j} = -\frac{\partial\overline{u'_J c'}}{\partial x_j} + \bar{s}\delta(x,0) \tag{2.9}$$

where $\delta(x, z)$ is the Dirac delta function. Everywhere except at the ground surface the advective flux is balanced by the divergence of turbulent flux and in the 1D situation, both are equal to zero.

We now let the flow be perturbed by any of the mechanisms we discussed in the introduction so that a steady 2D flowfield results from the addition of the perturbation to the original lD flow. Spatial variation in the mean flow inevitably results in the generation of spatial variation in the turbulent fluxes through four mechanisms. These have been analysed in an illuminating way by Raupach et al. (1992) in the context of flow over a low hill. They are:

1. Variation of radiant energy flux as a function of the angle the solar beam makes with the surface. This mechanism produces a streamwise variation in sensible and latent heat flux at the surface and, usually, in the flux of CO_2 as well. Obviously it is only important in complex topography but the next three mechanisms operate on flat ground as well as on hills.
2. Change of surface stress with x. This feeds into the scalar flux boundary condition. If we suppose a logarithmic description still to be valid at the surface, that is that if the flow is relatively slowly varying in x, then the scalar flux near the surface is described by a flux-gradient relationship with diffusivity ($\kappa u^* z$), κ being Von Karman's constant and u^* the friction velocity. It is the x dependence of the near-surface windfield that produces a streamwise variation in u^*.
3. Change of turbulent stresses in response to the 2D windfield. Spatial variation develops in the turbulent stresses in response to the 2D windfield. These in turn generate spatial variation in the eddy fluxes of the scalar as can easily be seen by considering the production terms of the relevant eddy flux rate equations, where the turbulent stresses multiply mean concentration gradients in terms that typically take the form, $\overline{u'_i u'_j}\partial\bar{c}/\partial x_j$ (Kaimal and Finnigan, 1994).
4. Changes in the mean scalar concentration field develop as isoconcentration lines (which were parallel to the ground in the lD case) are convected along the converging and diverging streamlines of the 2D flowfield. This 2D structure in mean concentration also feeds into the production terms of the eddy flux rate equations as seen in (3) above.

Even if the surface flux of c is maintained constant with x, the last three mechanisms set out above will ensure that horizontal variations will develop in $\overline{w'c'}(x, z_r)$ and in $\overline{u'c'}(x, z_r)$, invalidating assumption (2.5a).

2.2. Assumption (2.5b)

This is that the streamwise advection term $\bar{u}\partial\bar{c}/\partial x$ is negligible relative to the vertical advection $\bar{w}\partial\bar{c}/\partial z$ We can examine the consistency of this assumption best by a simple thought experiment. Remaining with the steady, 2D flowfield discussed above (Eq. (2.9)), we see that at any point in the interior of the flow, the divergence of eddy flux generated by mechanisms a-d must be balanced by the divergence of advective flux:

$$\bar{u}\frac{\partial\bar{c}}{\partial x} + \bar{w}\frac{\partial\bar{c}}{\partial z} = -\left(\frac{\partial\overline{u'c'}}{\partial x} + \frac{\partial\overline{w'c'}}{\partial z}\right) \tag{2.10}$$

The simplest conceivable situation is one where the imposed 2D flow is essentially inviscid and generates no eddy flux perturbation so the right hand side of (2.10) remains zero but $\partial\bar{c}/\partial x$ and $\partial\bar{c}/\partial z$ both change from their 1D values as the fluid is advected along the 2d streamlines (mechanism (4) of the last example). The terms $\bar{u}\partial\bar{c}/\partial x$ and $\bar{w}\partial\bar{c}/\partial z$ are now individually non-zero but, as the RHS of 2.10 is still zero, they must be equal and opposite everywhere in the flow.

Consider next a more realistic 2D flow, a perturbation of an originally 1D flowfield by convection cells that span the boundary layer. The unperturbed lD flowfield contained no mean streamwise velocity and convectively driven turbulence maintained the steady 1D flux of $\bar{c}(z)$. We impose upon this a set of symmetrical circulation cells generated by a succession of vortices of alternating sign, spaced regularly in the x direction and with their axes aligned in the y direction (Fig. 1). This flow pattern reproduces the essential elements of real, convectively generated, recirculating flows and, near the surface at least, we would expect a 2D pattern of eddy flux divergence to accompany the 2D mean flow.Ascending and descending stagnation streamlines ($\bar{u} = 0$) are located alternately between each pair of vortices. Directly above a vortex the flow is parallel to the ground, ($\bar{w} = 0$) (Fig. 1). On a vertical stagnation streamline the only component of the advective term is $\bar{w}\partial\bar{c}/\partial z$ Similarly, directly above a vortex, the only component of the advective flux divergence available to balance turbulent flux divergence is $\bar{u}\partial\bar{c}/\partial x$ In contrast to the inviscid case discussed above, where the eddy flux divergence remained zero, now, depending on where a tower is situated in the circulation cell, the vertical advection term can be responsible for all or none of the advective flux divergence and without prior knowledge of the 2D flowfield all we can say is that a measurement of $\bar{w}\partial\bar{c}/\partial z$ on a tower might represent some bound on the magnitude of the total advective flux divergence.

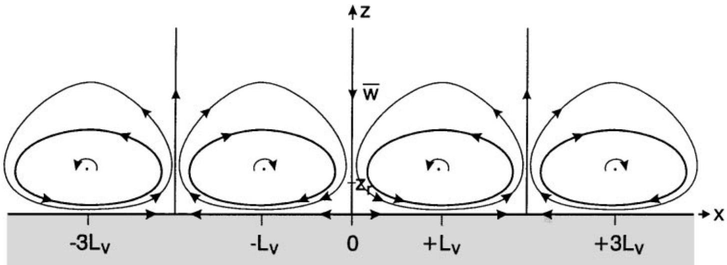

Fig. 1. Streamlines generated by a regular array of spanwise vortices.

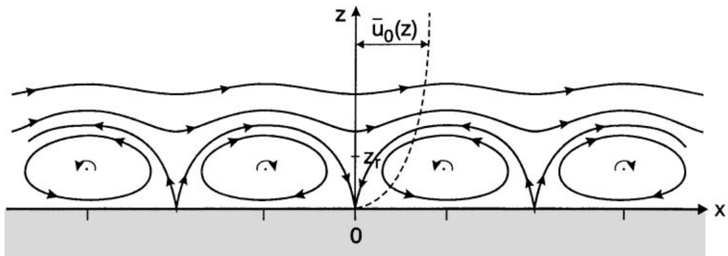

Fig. 2. Streamlines generated by the same vortex array with a mean streamwise velocity $\bar{u}_0(z)$.

If we now extend our model to the situation where the original 1D flow had a mean velocity component $\bar{u}_0(z)$, the mass balance becomes:

$$(\bar{u}_0 + \bar{u})\frac{\partial \bar{c}}{\partial x} + \bar{w}\frac{\partial \bar{c}}{\partial z} = -\left(\frac{\partial \overline{u'c'}}{\partial x} + \frac{\partial \overline{w'c'}}{\partial z}\right) \tag{2.11}$$

Adding a height dependent streamwise velocity $\bar{u}_0(z)$ to our simple model of alternating vortices means that stagnation streamlines are confined beneath some height that depends on the relative strengths of the vortices and the mean flow. Below this level the stagnation streamlines are vertical only at the surface (Fig. 2). Above the closed stagnation streamlines, the flowfield is a wave pattern with period 2L so that $\bar{w}$ passes through zero every L and $\bar{u}$ is never zero above $z=0$. In this flow, $\bar{w}\partial\bar{c}/\partial z$ can never account for all the advection term except at stagnation points, when both terms are zero anyway.

These two examples leave open the question of whether, in realistic 2D flows, there are universal relationships between the two components of advection that would allow us to use $\bar{w}\partial\bar{c}/\partial z$ as a bound on the total advection at any point in the flowfield. We can investigate this with a reasonable degree of generality by enlisting the model of Raupach et al. (1992) of scalar flux and concentration over a low hill. Although in this case the perturbing 2D flow field is caused by the presence of a hill we can view it as representative of any small perturbation whether it be caused by a hill or a weak manifestation of the diabatic effects we have already mentioned.

A fundamental feature of the linear model is the division of the flow domain into two parts: an inner region of depth l close to the surface, where both mean momentum and scalar fields are strongly affected by the changes in turbulent fluxes caused by the hill, and an outer region, $z > l$, where the mean flow responds inviscidly to the hill and changes to the scalar field are caused entirely by advection along the distorted mean flow streamlines. Because the model is linear we write Eq. (2.11) as:

$$\bar{u}_0\frac{\partial \bar{c}_1}{\partial x} + \bar{w}_1\frac{\partial \bar{c}_0}{\partial z} = -\frac{\partial \overline{w'c'}}{\partial z} \tag{2.12}$$

where subscript 0 denotes the undisturbed upwind variable and subscript 1 the perturbation caused by the hill and the model takes assumption (2.5c) as valid on the grounds of a rational scaling procedure. It is clear from (2.12) that the signs and relative magnitudes of the two advection terms depend on the sign of $\partial\bar{c}_0/\partial z$, which is independent of the presence of the hill, and on the way that $\partial\bar{c}_1/\partial z$ and $\bar{w}_1$ change as the flow goes over the hill.

Considering the horizontal advection term first, the linear theory computes $\bar{c}_1$ the perturbation to the concentration field (and hence $\bar{u}_0\partial\bar{c}_1/\partial x$ also) as the sum of four components, each the result of one of the four mechanisms we listed earlier as contributing to the change in eddy flux. These were: streamline convergence/divergence, changes in turbulent stresses, changes in surface shear stress and changes in surface scalar flux. As we traverse the hill, contributions to $\bar{c}_1$ from the first three of these vary in phase with each other. Below $z = l$ the effect of surface shear stress dominates, the influences on $\bar{c}_1$ of stress changes and streamline convergence/divergence being smaller in magnitude and opposing each other. Above $z = l$, only the streamline convergence/divergence effect is significant. The fourth contribution to $\bar{c}_1$, that from changes in surface scalar flux, depends entirely upon how this changes as we pass over the hill and need not be in phase with the other contributions to $\bar{c}_1$. For evaporation and sensible heat especially, it will respond to the interaction of slope angle and sun elevation and to the difference of water availability between hilltop and valley.

Moving now to the vertical advection term, $\bar{w}_1\partial\bar{c}_0/\partial z$, the theory shows that over the hill, $\bar{w}_1$ varies in phase with the first three contributions to $\bar{u}_0\partial\bar{c}_1/\partial x$. Furthermore, within the assumptions of linearity, contributions to $\bar{u}_0\partial\bar{c}_1/\partial x$ from changes in turbulent stresses and changes in surface shear stress do not depend directly on the sign of $\partial\bar{c}_0/\partial z$ so that changing the sign of the background scalar gradient, as occurs for example during the twice-daily switch between assimilation and respiration dominance of the CO_2 flux, can reverse the sign of the vertical advection term in the inner layer with only a small effect on the horizontal one.

With these results we are in a position to compare the relative signs and magnitudes of the two advection terms. Above the inner layer, where only the effect of streamline convergence/divergence affects the scalar perturbation, the theory yields the simple result that the two advection terms vary in phase but are of opposite sign and approximately equal magnitude This is the result we would have expected from our earlier discussion of a general inviscid 2D perturbation. In the inner layer, where most measurements are made, the sign of the horizontal advection term $\bar{u}\partial\bar{c}_1/\partial x$ is determined by the relative strengths of the four competing influences on $\bar{c}_1$ although with uniform surface roughness and constant surface scalar flux, total $\bar{u}_0\partial\bar{c}_1/\partial x$ will have the same sign as its contribution from streamline convergence/divergence but will be of considerably larger magnitude (Raupach et al., 1992). In the inner layer, the sign and magnitude of the vertical advection term is set by the background scalar gradient in a way that is essentially independent of the dominant contributions to the horizontal term.

In summary, according to this simple linear model, which we might have expected to indicate any universal regularities between the two advection terms, the two contributions to advection are almost equal and opposite in regions of the flow, where changes in turbulent flux divergence are negligible. Within the inner layer, where most flux measurements will be made, the two terms may be of considerably different magnitude and the same or opposite sign, depending on the relative contributions of the various competing influences on $\bar{c}_1$. Clearly we have no grounds for using a measurement of $\bar{w}\partial\bar{c}/\partial z$ as a bound on total advection at an arbitrary point in the flow.

Finally let us relax the conditions of our thought experiment still further and allow the flow to be 2D and unsteady. This allows us to consider the passage of cells of the convective boundary layer which generate 2D (in reality, 3D) flow fields as they pass over the tower. Eq. (2.11) becomes:

$$\frac{\partial\bar{c}}{\partial t} + (\bar{u}_0 + \bar{u})\frac{\partial\bar{c}}{\partial x} + \bar{w}\frac{\partial\bar{c}}{\partial z} = -\left(\frac{\partial\overline{u'c'}}{\partial x} + \frac{\partial\overline{u'c'}}{\partial z}\right) \tag{2.13}$$

and the question of the relative importance of the three components of the total derivative, the LHS of (2.13), becomes one of timing. We can represent the passage of convection cells simply by translating the x axis in the first example (Fig. 1.) at a velocity -U_c, relative to the tower, where U_c is the passage velocity of the cells. We obtain exactly the same result as in the last two examples except that the alternating regions of ($\bar{u} = 0$, $\bar{w} = 0$) convect past the tower with a frequency $U_c/2L_v$,where L_v is the streamwise spacing of the vortices (Fig. 2).

To summarize, assumption (2.5b) is not a valid statement of mass conservation except in the fortui-

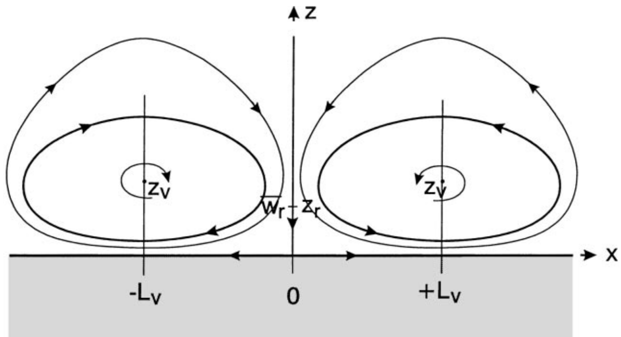

Fig. 3. Flow generted by a pair of spanwise vortices.

tous circumstance that the tower is located on the stagnation streamline of a spatially fixed flow pattern without a large scale background mean velocity.

2.3. Assumption (2.5c)

Unfortunately, we cannot use arguments of comparable simplicity to decide whether $\partial\overline{u'c'}/\partial x$ can always be neglected relative to $\partial\overline{w'c'}/\partial z$ This is because the turbulent flux field generated by even a simple 2D mean flow is the result of complex interactions not easily represented by a thought experiment. We may once again seek guidance in a specific example by resorting to the model of Raupach et al. (1992) of scalar fields and fluxes over a low hill. This linearised approach represented the surface normal flux via an eddy viscosity and was able to ignore the horizontal eddy flux divergence by appealing to a consistent scaling procedure. We might cautiously conclude from this that assumption (2.5c) is justified for measurements close to the ground in 2D motions weak compared to the 1D background flow.

2.4. Assumption (2.5d)

We recall that this is that we can write $\partial\bar{w}/\partial z \cong \bar{w}_r/z_r$ and so simplify the integration by parts in Eq. (2.8). Since $\bar{w}(z)$ must vanish at the ground and equal $\bar{w}_r$ at z_r the assumption is correct in order of magnitude but the height dependence of $\partial\bar{w}/\partial z$ might still be significantly different from linear. We can only test this by assuming a model for the 2D airflow pattern that gives rise to $\bar{w}(z)$. Since we have already concluded that Eq. (2.8) is only valid near a vertical stagnation streamline, we will consider the downdraft generated between an isolated pair of the fixed, counter−rotating vortices of Fig. 1. The vortex centres are located at ($xs = \pm L$, $z = z_v$) (See Fig. 3). Assuming inviscid flow, classical hydrodynamic theory (e.g. Milne-Thompson, 1968) gives the vertical velocity gradient on the stagnation streamline through $x = 0$ as

$$\frac{\partial\bar{w}}{\partial z} = \bar{w}_r \frac{\left[\left(1/\left(L_v^2+(z-z_v)^2\right)\right) - 2(z-z_v)^2/\left(L_v^2+(z-z_v)^2\right)^2 + \left(1/\left(L_v^2+(z+z_v)^2\right)\right) - 2(z+z_v)^2/\left(L_v^2+(z+z_v)^2\right)^2\right]}{\left[(z-z_v)/\left(L_v^2+(z-z_v)^2\right) + (z+z_v)/\left(L_v^2+(z+z_v)^2\right)\right]} \tag{2.14}$$

and for $L_v \gg z$, z_r, z_v, (2.13) reduces to $\partial\bar{w}/\partial z \cong \bar{w}_r/z_r$ as assumed by Lee. More complicated and realistic flow field models produce essentially the same conclusion: as long as the horizontal scale of the 2D flow pattern is much larger than the reference height, then $\partial\bar{w}/\partial z$ is approximately constant between z_r and the ground.

2.5. Coordinate systems and mean vertical velocities

Lee suggests a practical procedure for defining the mean vertical velocity using a two stage process. The procedure is a response to the experimental difficulties of measuring a vertical velocity which is typically not much larger than error terms caused by sonic anemometer misalignment or transducer shadowing. It consists in regressing vertical velocity measured in the reference frame of the sonic against the azimuth of the mean velocity. The regression employs all the measuring periods available. The least squares fit of the vertical velocity against azimuth is taken to be the vertical velocity forced by the underlying terrain, viz:

$$w_{\text{measured}} = \bar{w} + \hat{w} \quad \text{and} \quad \hat{w} = a(\phi) + b(\phi)\hat{u} \tag{2.15}$$

where $\hat{w}$ is the component of vertical velocity assumed to be forced by the underlying terrain and $\hat{w}$ is the departure of the vertical velocity measured in any averaging period from this value; $\hat{u}$ is the measured total horizontal velocity and ϕ is the azimuth angle. (I have departed from Lee's notation slightly in writing 2.15.) As pointed out by Lee, the procedure requires $\bar{w}$ to be distributed randomly about $\hat{w}$ to avoid bias. We note that, if $\hat{w}$ is really to represent a terrain-forced velocity, it would be advisable to include in the regression sample only measuring periods when the flow was near neutral stratification, i.e., times of strong wind

If readers are to use this procedure the following two points are salient

1. The triple $\hat{u}, \hat{w}, \phi$ can be used to define the axes of a Cartesian coordinate system $\hat{X}_i$ aligned with the local long-term mean streamline as defined by the regression procedure. This coordinate frame is not necessarily parallel to the underlying terrain, in fact in complex terrain it will usually not be so.
2. If we wish to use the departure value of mean velocity, $\bar{w}$ in Eq. (2.2), then we are, by default, choosing to use $\hat{X}_i$ coordinates as the basis for analysis and must rotate all the other terms appearing in (2.2) and equations derived from it into the $\hat{X}_i$ frame. If it should occur that $\bar{w}/\bar{u}$ is significant (eg. very light winds and anabatic flows), then the changes to the other terms in (2.2) may well be of the same order as any calculated advection correction.

Rotating into the Cartesian frame $\hat{X}_i$, aligned with the long-term mean streamlines is not equivalent to working in streamline coordinates. In streamline coordinates the mean velocity always defines the x axis; there are no mean velocity components in the y and z directions. In two dimensions the streamline coordinate version of the conservation Eq. (2.2) is:

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\partial_x \bar{c} = -\partial_x \overline{u'c'} - \frac{\overline{u'c'}}{L_a} - \partial_z \overline{w'c'} + \frac{\overline{w'c'}}{R} + \bar{s}(x,z) \tag{2.16}$$

where

$$\frac{1}{L_a} = \frac{1}{\bar{u}}\partial_x \bar{u} \quad \text{and} \quad \frac{1}{R}$$

is the local radius of curvature of the streamline.

The advantage of using streamline coordinates is that the advective term is considerably simplified but at the expense of the extra compensating terms in the turbulent flux divergence. Not surprisingly, these extra terms can be of the same order as the advective terms that they replace. Note that in (2.16) partial derivatives have been replaced by directional derivatives denoted by *(*∂_x*,* ∂_z*)*.The only practical consequence of this in most cases is that, when manipulating Eq. (2.16), the order of multiple differentiations or integrations cannot be arbitrarily changed. More information on streamline coordinates and their use may be found in Kaimal and Finnigan (1994) and references therein.

3. Discussion and conclusions

1. The kind of advective flow fields that give rise to the mean vertical velocities discussed by Lee (1998) are intrinsically two- or three-dimensional and should be analysed as such. This requires us to integrate the point valued conservation equation horizontally as well as vertically when we set out to relate net ecosystem exchange to atmospheric fluxes measured by instruments on towers above the vegetation.
2. Perturbing an initially 1D flow to produce a 2D or 3D flow field results in a corresponding 2D or 3D pattern in the turbulent fluxes of scalars. However, adopting a lD analysis framework (Eq. (2.6)) in a 2D or 3D flow carries the implied assumption that the eddy fluxes measured on the tower are equal to their horizontal average values over the control volume or 'footprint' While the magnitude of their departures is impossible to test without measurements or a model of the flow and concentration field, this assumption does not hold in general.

3. The 2D or 3D pattern of eddy flux divergence must be balanced in the interior of the flow by an advective flux divergence: $\bar{u}\partial\bar{c}/\partial x + \bar{w}\partial\bar{c}/\partial z$. We can show by examining the mean velocity fields of simple recirculating 2D flow fields that the vertical advection term is responsible for all the advective flux divergence only in the particular circumstance that measurements are made on a vertical stagnation streamline. Such a streamline cannot exist when the 2D or 3D perturbation is superimposed on a lD flow driven by a steady, height dependent mean velocity.
4. We have examined the question of whether, in the case of a flow with a large scale background wind, there are any universal regularities in the relationship of the two advection terms that would allow us to view the NEE obtained by applying Lee's correction as a bound on the possible value. Using a linear model of scalar flow and transport over a low hill as a test case, we conclude that above the region where perturbations to the turbulent flux divergence are significant, the two advection terms are essentially equal and opposite so including only one of them in a budget calculation will actually make things worse. Closer to the surface in the region where most measurements are made, the signs and relative magnitudes of the two terms are determined by a set of competing influences that are specific to a particular flow and including only the vertical advection term in the budget cannot be justified.
5. It is impossible to test the assumption that the horizontal divergence of turbulent flux is small relative to the vertical, i.e., $\partial\overline{u'c'}/\partial x \ll \partial\overline{w'c'}/\partial z$ except in relation to a particular flow field. Comparison with a small perturbation model of scalar fluxes over a low hill suggests, however, that this is a reasonable assumption when the measurement height is much smaller than the horizontal scale of the 2D or 3D motion.
6. Simple inviscid flow theory enables us to test the third of Lee's explicit assumptions without adopting the full panoply of a model for the distorted concentration field. We conclude that it is valid to assume that $\partial\bar{w}/\partial z \cong \bar{w}_r \cong \bar{w}_r/z_r$ as long as the horizontal scale of the two- or three-dimensional flow field generating $\bar{w}_r$ is much larger than z_r.
7. We also note that practitioners must take care, when defining the mean vertical velocity relative to a chosen coordinate frame, that they also rotate all the other terms in the conservation equation into the chosen coordinate frame. The chosen frame might be the Cartesian frame defined by the regression of the long term data set against azimuth, that is, essentially, the Cartesian frame aligned with the long term mean streamlines. We point out also that this does not amount to working in 'streamline coordinates'.

It is reasonable to ask, therefore, why the advective correction proposed by Lee (that is the retention of $\bar{w}\partial\bar{c}/\partial z$ only) appears to improve energy and carbon budget closures at some sites (Lee, 1998, Dr D. Baldocchi, pers. commun.). It may be that at these sites, towers are situated on ridges or hills which may be acting as foci for diabatic downdrafts or updrafts so that the measuring position is indeed located at or near a stagnation streamline at times of light wind when the advective correction might be most important.

We should not expect to be able to use simple universal corrections to close scalar budgets in 2D or 3D flow fields, when we only have measurements at a single point. The alternatives are to work at a topographically ideal site and accept the intrinsic limits to accuracy imposed by the stochastic character of atmospheric flow or, if one's site is less than ideal, undertake the modelling and more intensive measurements needed to characterise the flow patterns peculiar to that site, see for example Sun et al. (1998). Given the scale of the investment that a multi-year eddy flux experiment represents, this latter course may be attractive for sites that are topographically complex.

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